Statistical Modeling and Analysis of Hardness Value Change with Diametric Distance of Heat Treated Steel

Aondona Paul Ihom, Aniekan Offiong

Abstract— The work 'Statistical Modeling and Analysis of Hardness Value Change with Diametric Distance of Heat Treated Steel' has been carefully undertaken using data generated empirically at the University of Jos, Makurdi Campus, Department of Materials and Metallurgical Engineering heat treatment laboratory. A large data was generated through the carburization of carburizing steel using 80 wt% charcoal/ 20 wt. % cowbone. The operation was carried out at 900OC for 6 hrs using electric furnace. The quenching was done in water and tempering was done at 150OC. The data was subjected to statistical analysis and it was discovered that there was strong relationship between the hardness and the distance below the surface of the carburized steel. This was reflected in the coefficient of determination (r2) which was +0.98. A mathematical model was developed for the prediction of hardness at any point below the surface of the heat treated steel and the accuracy of the model was determined using various parameters like standard error, confidence limits and significance tests.

Index Terms— Statistical modeling, Analysis, Hardness, Steel, heat treatment, Temperature and Distance.

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1 Introduction

Carburized steels are designated for the production of parts and components which are subjected to the combined action of impact load and wear. They should therefore have high hardness and resistance to wear. This is achieved by the process of concentration i.e saturation of the steel surface with carbon. For this purpose low carbon steels are chosen e.g. steel containing 0.2% C/ Cr, 0.2%C/Cr/Mg, 0.2%C/Cr/Mn, 0.25%C/Cr/Mg/Ti, 0.3%C/Cr/Mg/Ti, in order to achieve the final properties of carbon steels, they are heated specifically to austenite temperature and quenched. After which they are given low temperature tempering at 150^{o}C -200^{o}C [1]. The study of the hardness pattern of heat treated steel has been of great interest, over the years many researchers [1],[2],[3] have carried out research work in demonstrating the hardenability of steel after quenching, one of such popular tests are the grossmann hardenability test and quench hardenability test bar which indicates the reduction in hardness as measurements are taken towards unquenched portion of the test bar. The same pattern is observed in carburized steel which has a hard case and a soft core [4],[5], [6],[7]. It is interesting to note that statistical approach and mathematical modeling can be used to exhaustively study the relationship of the hardness of carburized steel which has been quenched and tempered in relation to distance variation towards the core of the heat treated steel [8], [9].

The developed mathematical model can then be used to predict the hardness of the carburized steel at any distance below the surface of the steel. Such models can be of great assistance when dealing with such steels for the purpose of hardness estimation. Similar works have been carried out by several researchers [8],[9],[10]. Nwoye, et al [11] used statistical models in the study of desulphurization limit of Agbaja ore. The work use statistical analysis in the study of the variables and also developed models for predicting of desulphurization limit. Previous works have all proven that mathematical models and statistical analysis are invaluable for prediction and the study of relationships [8],[9],[10], [11],[13].

The objective of the work is to undertake the statistical modeling and analysis of hardness value change with diametric distance of heat treated steel.

2 Materials And Method

2.1 Materials

The materials used for this work included charcoal cowbone, carburizing steel, acetone, water, and clay. The equipment that were used included electric muffle furnace made by +GF of Germany, Vickers Microhardness testing machine model MHT-1 No: 8331 made by Matsuzawa Seiki Co.Ltd., of Japan. Others included grinding and polishing machines and specimen lathe and power hack saw.

2.2 Method

The method adopted in the laboratory was to carry out the carburization process for 6 hrs at 900^{o}C using carburizing boxes which were filled with 80 wt. % charcoal/ 20 wt. % cowbone as carburizing material. The furnace had a temperature sensitivity of ±5^{O}C. The quenching was done in water. The specimens were immediately tempered at 150^{o}C for 1 hr. The specimens were then prepared for testing, using the equipment mentioned above, the data generated in some of the test is what is used in this work. The data was generated empirically as a result of several tests in the heat treatment laboratory of university of Jos, Makurdi Campus-Nigeria.

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3 RESULTS AND STATISTICAL ANALYSIS

1.1 Results

The result of the work is shown in Table 1 which is the variation of average hardness values in Hv with Distance below surface of heat treated steel.

TABLE 1

VARIATION OF AVERAGE HARDNESS VALUES IN HV WITH DISTANCE BELOW SURFACE OF THE HEAT TREATED STEEL

S/No	Distance below Surface (mm)	Average Hardness Value (mm)
1.	0.2	960
2.	0.4	912
3	0.6	866
4	0.8	823
5	1.0	782
6	1.2	783
7	1.4	706
8	1.6	670
9	1.8	636
10	2.0	605
11	2.2	575
12	2.4	546
13	2.6	518
14	2.8	492
15	3.0	468
16	3.2	448
17	3.4	422
18	3.6	401
19	3.8	381
20	4.0	362

3.2 Statistical analysis

Twenty measurements of hardness values were taken and for each measurement the average value was taken as the distance was varied towards the core of the heat treated steel at a constant interval of 0.2mm. This statistical analysis will help us to see the relationship between the hardness and the variation in distance between the surface of the carburized steel and and the core of the steel and to be able to model the relationship for prediction of hardness values below steel surface after carburizing operation. The analysis is wholly dependent on prevailing conditions as stated in the method.

Calculating the values of a and b

In the general form of the equation for a straight line

$Y = a + bx$ (1)

Where, a and b are constants and a represents the fixed element, and b the slope of the line.

To find a and b it is necessary to solve two simultaneous equations known as the` normal equations'

Which are:

$an + b\sum x = \sum y$ (2)

$a\sum x + b\sum x^2 = \sum xy$ (3)

where,

n = number of pairs of figures

The data below is used

TABLE 2

DERIVED FROM DATA IN TABLE 1

S/No	Distance below Surface X(mm)	Hard-ness (Hv) Y	X^2	Y^2	XY
1.	0.2	960	0.04	921600	192.0
2.	0.4	912	0.16	831744	364.8
3.	0.6	866	0.36	749956	519.6
4.	0.8	823	0.64	677329	658.4
5.	1.0	782	1.0	611524	782.0
6.	1.2	783	1.44	613089	939.6
7.	1.4	706	1.96	498436	988.4
8.	1.6	670	2.56	448900	1072.0
9.	1.8	636	3.24	404496	1144.8
10.	2.0	605	4.00	366025	1210.0
11.	2.2	575	4.84	330625	1265.0
12.	2.4	546	5.76	298116	1310.4
13.	2.6	518	6.76	268324	1346.8
14.	2.8	492	7.84	242064	1377.6
15.	3.0	468	9.00	219024	1404.0
16.	3.2	448	10.29	200704	1433.6
17.	3.4	422	11.56	178084	1434.8
18.	3.6	401	12.96	160801	1443.6
19.	3.8	381	14.44	145161	1447.8
20.	4.0	362	16.00	131044	1448.0
$\sum$	42	12356	114.8	8297046	21783.2

$20a + 42b = 12356$ (4)

$42a + 114.8b = 21783$ (5)

Solving these two equations simultaneously we have

$a = 946.57$ and $b = -156.56$

Therefore the regression line is obtained by substituting the values in equation 1

$y = 946.57 - 156.56x$ (6)

Using the results of the regression analysis

The above formula, equation (6) can now be used to predict the hardness value in Hv at any given depth e.g 4.2mm towards the core of the heat treated steel.

Let y = H = hardness, x = distance below the surface.

H = 946.57 – 156.56x4.2 = 289Hv

3.21 Accuracy of the Developed Model

Coefficient of determination (r^2)

- *Dr. Aondona Paul Ihom is a Lecturer with University of Uyo, Department of Mechanical Engineering, PMB 1017 Uyo, Akwaibom State-Nigeria*
- **Corresponding Author's Email**: paulihom@yahoo.co.uk; **Tel: (+2348035813571)**
- *Dr. Aniekan Offiong is a Lecturer with* University of Uyo, Department of Mechanical Engineering, PMB 1017 Uyo, Akwaibom State-Nigeria
- ***Coauthor's Email:*** *irimanagement @yahoo.com, Tel: +2347032254829*

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$$r^2 = \frac{\text{Explained variation}}{\text{Total Variation}} = \frac{\Sigma(yE - \hat{y})^2}{\Sigma(y - \hat{y})^2} \quad (7)$$

Where, YE = estimate of y given by the regression equation for each value of x.

Ÿ = mean of actual values of y

Y = individual actual values of y

r^2 will be calculated for the data in Table 1 for which the regression line is shown in equation 6

$$Ÿ = \frac{12356}{20} = 617.8$$

TABLE 3

CALCULATION OF COEFFICIENT OF DETERMINATION (R^2)

X	y	yE	YE- Ÿ	(yE- Ÿ)^2	y - Ÿ	(y- Ÿ)^2
0.2	960	915.25	297.45	88476.50	342.2	117100.84
0.4	912	883.95	266.15	70835.82	294.2	86553.64
0.6	866	852.63	234.83	55145.13	248.2	61603.24
0.8	823	821.32	203.52	41420.39	205.2	42107.04
1.0	782	790.01	172.21	29656.28	164.2	26961.64
1.2	783	758.70	140.90	19852.81	165.2	27291.04
1.4	706	727.39	109.59	12009.97	88.2	7779.24
1.6	670	696.07	78.27	6126.19	52.2	2724.84
1.8	636	664.76	46.96	2205.24	18.2	331.24
2.0	605	633.45	16.65	277.22	-12.8	163.84
2.2	575	602.14	-15.66	245.24	-42.8	1831.84
2.4	546	570.83	-46.97	2206.18	-71.8	5155.24
2.6	518	539.51	-78.29	6129.32	-99.8	9960.04
2.8	492	508.20	-109.60	12012.16	-125.8	15825.64
3.0	468	476.89	-140.91	19855.63	-149.8	22440.04
3.2	448	445.58	-172.22	29659.73	-169.8	28832.04
3.4	422	414.27	-203.53	41424.46	-195.8	38337.64
3.6	401	382.95	-234.85	55154.52	-216.8	47002.24
3.8	381	351.64	-266.16	70841.15	-236.8	56074.24
4.0	362	320.33	-297.47	88488.40	-255.8	65433.64
∑	12356			652022.34		663509.2

$$r^2 = \frac{652022.34}{663509.20} = 0.98 \quad \therefore \ r^2 = 98\%$$

This result may be interpreted that 98% of the hardness variation towards the core of the heat treated steel depend on the variation of the distance below the surface of the heat treated steel. Other factors account for only 2% of the variation in hardness of the heat treated steel. r^2 is very high which means there is a strong relationship between distance below the surface of the cast hardened steel and hardness value. This result agrees with earlier studies by several authors [12],[13],[14],[15],[16],[17]. It also implies that the relationship is linear.

Standard error of regression

$$\text{Standard error of regression} = Se = \frac{\sqrt{\Sigma y^2 - a\Sigma y - b\Sigma xy}}{n - 2} \quad \textbf{(8)}$$

This standard error is also known as the residual standard deviation. From table 2 our residual standard deviation is given as

$$Se = \sqrt{\frac{\sqrt{8297046 - 946.57 \times 12356 - (-156.56) \times 21783.2}}{20 - 2}} = 25.39$$

This value is used below in setting confidence limits for the calculated regression line.

Confidence limits

The confidence limits for the whole of the regression line are calculate by using a quality known as the standard error of the average forecast which is given by

$$S_{ef} = Se\sqrt{1/n + (x\text{-}X)^2/\ \Sigma x^2 - (\Sigma x)^2/\ n} \quad (9)$$

Although in a given problem Se will be a constant amount, the expression

$\sqrt{1/n + (x\text{-}X)^2/\ \Sigma x^2 - (\Sigma x)^2/\ n}$ will vary according to the value of x. The calculations for x = 0.2 to 4 are given below

The expressions $1/n$ and $\Sigma x^2 - (\Sigma x)^2/\ n$ are both constant for a given problem.

It is $(x - \bar{x})^2$ which will vary and must be calculated for each value of x

$1/n = 1/\ 20 = 0.05$

$\Sigma x^2 - (\Sigma x)^2/\ n = 114.8 - 42^2/\ 20 = 26.6$

$X = 42/\ 20 = 2.1$

TABLE 4

VARIATION OF THE EXPRESSION (x- X)^2

X	(x-X)^2
0.2	3.61
0.4	2.89
0.6	2.25
0.8	1.69
1.0	1.21
1.2	0.81
1.4	0.49
1.6	0.25
1.8	0.09
2.0	0.01
2.2	0.01
2.4	0.09
2.6	0.25
2.8	0.49
3.0	0.81
3.2	1.21
3.4	1.69
3.6	2.25
3.8	2.89
4.0	3.61

The expression $\sqrt{1/n + (x\text{-}X)^2/\ \Sigma x^2 - (\Sigma x)^2/\ n}$ may now be evaluated

When x = 0.2

$\sqrt{1/20 + 3.61/26.6} = 0.44$

In a similar fashion the other values are calculated and the results tabulated

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TABLE 5

VARIATION OF THE VALUES OF X WITH THE EXPRESSION $\sqrt{1/n + (x-X)^2 / \Sigma x^2 - (\Sigma x)^2 / n}$

x	Value of $\sqrt{1/n + (x-X)^2 / \Sigma x^2 - (\Sigma x)^2 / n}$
0.2	0.44
0.4	0.40
0.6	0.37
0.8	0.34
1.0	0.31
1.2	0.28
1.4	0.26
1.6	0.24
1.8	0.23
2.0	0.225
2.2	0.225
2.4	0.23
2.6	0.24
2.8	0.26
3.0	0.28
3.2	0.31
3.4	0.34
3.6	0.37
3.8	0.40
4.0	0.44

These values are used in the calculation of confidence intervals

Confidence interval

t distribution is used, n -2 = degrees of freedom, 20-2 = 18 degrees of freedom

The interval is calculated by estimating the fitted value of y for each value of x in the original data using the equation y = a + bx.

The interval then takes the form $y \pm S_{ef} \times t$

a = 946.57, b =- 156.56, Se = 25.39

$S_{ef} = Se\sqrt{1/n + (x-X)^2 / \Sigma x^2 - (\Sigma x)^2 / n}$ $= 25.39 \times value\ from\ table\ 5\ above$

t = 2.101 for 18 degree of freedom and a 95% confidence interval. The confidence interval can be now calculated as follows:

When x = 0.2, y = 946.57 – 156.56(0.2) = 915.26, the limits round this estimates are

$915.26 \pm 2.101 \times 25.39 \times 0.44 == 915.26 \mp 23.47$ This gives an upper limit of 938.73 and lower limit of 891.79

This process is repeated for the other values of x resulting in the following summary table, Table 6 suitably rounded.

TABLE 6

CONFIDENCE INTERVAL FOR THE RESULTS OF THE REGRESSION LINE (THE MODELED EQUATION)

		Confidence interval	
X	**Y**	**Lower limit**	**Upper limit**
0.2	915.26	891.79	938.73
0.4	883.95	862.61	905.29
0.6	852.63	832.89	872.37
0.8	821.32	803.18	839.46
1.0	790.01	773.47	806.55
1.2	758.70	743.76	773.64
1.4	727.39	713.52	741.26
1.6	696.07	683.27	708.87
1.8	664.76	652.49	677.03
2.0	633.45	621.45	645.45
2.2	602.14	621.45	645.45
2.4	570.83	558.56	583.10
2.6	539.51	526.71	552.31
2.8	508.20	494.33	522.07
3.0	476.89	461.95	491.83
3.2	445.58	429.04	462.12
3.4	414.27	396.13	432.41
3.6	382.95	363.21	402.69
3.8	351.64	330.30	372.98
4.0	320.33	296.86	343.80

Confidence interval for individual predictions standard error of the individual forecast

S_{ef} (individual) = $Se\sqrt{1 + 1/n + (x-X)^2 / \Sigma x^2 - (\Sigma x)^2 / n}$

using the data in Table (2) and a value of x= 4.2 mm the above formula can be demonstrated

S_{ef} (individual) = $25.39\sqrt{1+ 1/20 + (4.2-2.1)/ 26.6} = 28$

When x = 4.2, y = 289 which has individual confidence intervals of $289 \pm 2.101 \times 28$ giving a lower limit of 230.17 and a upper limit of 347.83

These values should be contrasted with the value obtained for the general confidence interval in Table 6 which when an individual prediction of y is made the confidence intervals are much wider

Standard Errors of the intercept (a) and the gradient b

Standard error for intercept $S_a = Se\sqrt{\Sigma x^2 / n\Sigma x^2 - (\Sigma x)^2}$

Standard error for gradient $S_b = Se / \sqrt{\Sigma x^2 - (\Sigma x)^2 / n}$

Where Se is the standard error of regression

The confidence interval for α and β may be established as follows:

For the intercept $\alpha = a \pm t \times Sa$

For the gradient $\beta = b \pm t \times Sb$

The value of t is based upon n-2 degrees of freedom and the chosen confidence level

α is the population value for the intercept

β is the population value for the gradient

It is possible to construct a test of significance for α and β

For the intercept

H_O: $\propto = Some\ chosen\ value$
H_1 : $\alpha \neq Some\ chosen\ value$

The test statistics is t = a- α /S_a

For the gradient
H_O: $\beta = 0$
H_1: $\beta \neq 0$
The test statistics is t = b – β/ S_b

In both cases, the calculated value of t is compared with the tabulated value for n-2 degrees of freedom at the chosen level of significance
Using the formula
N =20, a =946.57, b = -156.56, t = 2.101
$\sum_x 2 = 114.8, \sum x = 42, \quad Se = 25.39$

The standard error of the intercept = S_a = 25.39√ 114.8/ 20 x 114.8-42^2 = 11.80

The 95% confidence interval of the intercept is
$\propto = 946.57 \pm 2.101 \times 11.80 = 946.57 \mp 24.78$ which gives an upper limit of 971.35 and a lower limit of 921.79
Significance test for the intercept
H_O: $\propto = 0$
H_1: $\propto \neq 0$
t = a- α /S_a = 946·57- 0/ 11.80 = 80.22

Since 80.22 is much greater than 2.101 (the value from t tables) H_O can be rejected.
Standard error of the slope = S_b = 25.39/ √114.8 - 42^2 /20 = 4.92
The 95% confidence interval for the slope is
$\beta = -156.56 \mp 2.101 \times 4.92 = -156 \pm 10.34$
Giving an upper limit of – 146.22 and lower limit of – 166.90
Significance test for slope
H_O: $\beta = 0$
H_1: $\beta \neq 0$
t = b – β/ S_b = 156.56 – 0/ 4.92 = 31.82

Since 31.82 > 2.101, H_O can be rejected. On the basis of this evidence the regression equation
$y = 946.57 - 156.56x$, this equation could be use as a basis for predicting hardness measurements taken towards the core of case hardened steel. The presentation of the statistical analysis of the result of the variation of the hardness of the heat treated steel from the surface to the core completely agrees with the theories as presented in references cited in this work [11],[12],[13],[14],[15],[16],[17].

4 CONCLUSION

The work 'Statistical Modeling and Analysis of Hardness Value Change with Diametric Distance of Heat Treated Steel' has been carried out. The outcome of the work is as follows:

1. On the basis of the above evidence the regression equation $y = 946.57 - 156.56x$ could be used as a basis of prediction of hardness measurements taken towards the core of case hardened steel, provided the conditions are the same with those mentioned in the method.
2. Result showed a very strong positive correlation between the hardness and the distance below surface
3. The coefficient of determination (r^2) had a strong positive value of 0.98, indicating that 98% of the hardness variation towards the core of the steel depends on the variation of distance below surface. Other factors account for only 2%.
4. The accuracy of the model equation has been gauged by the residual standard of deviation, standard error, and the confidence limit, including significance tests.

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REFERENCES

[1] D.A. Okongwu, ``Assessment of the Efficacy of Some Carbonate Minerals as Energizers in Pack Carburization of Mild Steel,'' *J. NSE*, 30-35, 1989.

[2] E.E. Nnuka, ``Heat Treatment,'' Lecture presented in the Department of Materials and Metallurgical Engineering, University of Jos, Makurdi Campus, 1991.

[3] A.P.Ihom, G.B.Nyior, I.J. Nor, N.J.Ogbodo, ''Investigation of Egg Shell Waste as an Enhancer in the Carburisation of Mild Steel,'' *American Journal of Material Science and Engineering* (1) 2, pp29-33, 2013.

[4] F.O.Aramide, A. I.Simeon , O.O.Isiaka and O. B. Joseph, ''Pack Carburization of Mild Steel, using Pulverized Bone as Carburize Optimizing Process Parameters,'' *Leonardo Electronic Journal of Practices and Technologies* ISSN 1583-1078 Issue 16, pp1-12, 2010.

[5] A.P.Ihom, S.A.Yaro, V.S.Aigbodion, ''The Effect of Carburisation on the Corrosion Resistance of Mild Steel in Four Different Media,'' *Journal of Corrosion Science and Technology*, 3, pp18-21, 2005.

[6] A.P.Ihom, ''Case Hardening of Mild Steeling using Cow-bones,'' B.ENG Degree Project Submitted to the Department of Materials and Metallurgical Engineering, University of Jos,1991.

[7] A.P.Ihom, G.B. Nyior, and M.Ambayin, ''Surface Hardness Improvement of Mild Carbon Steel using Arecaceae Waste Flower Droppings,'' the Pacific Journal of Science and Technology (13), 1, pp133-138, 2012.

[8] T.Lucey, *Quantitative Techniques*, 7th Edition Low Priced Edition Funded by the British Government, pp99-150, 2000.

[9] W.G.Cochran, and P.V.Youle, *Experimental Design*, 2nd Edition Wiley, New-york, pp30-45, 1957.

[10] Y.P.Adler, E.V.Markova, and Y.V.Granovshy, *The Design of Experiments to Find Optimal Conditions*, Mir Publishers, Moscow, pp25-50, 1975.

ISSN 2229-5518

[11] C.I.Nwoye, C.U. Atuanya, and J.U.,Uroha, ''Statistical Models in the Study of Desulphurization Limit of Agbaja Ore, University of Usak,'' *Journal of Material Science and Engineering*, 5, pp34-38, 2013.

[12] R.A. Higgins, *Properties of Engineering Materials* 6th edition Hodder and Stoughton Educational Great Britain, pp199-200, 1983.

[13] ASM ''Committee on Gas Carburizing,'' American Society for Metals, USA, pp1-46, 1977

[14] A,P.Ihom,G.B.Nyior, M.U.Suleiman, and G.Z.Ibrahim, ''Improving Surface Hardness of Steel using Rice Husk Waste,'' *Nigerian Journal of Tropical Engineering*, (1)2, pp107-115, 2011.

[15] A.P.Ihom, G.B.Nyior, O.O.Alabi, S.Segun, I.J.Nor, and J.N.Ogbodo, ''The Potentials of Waste Organic Materials for Surface Hardness Improvement of Mild Steel,'' *International Journal of Scientific and Engineering Research*, (3).11, pp1-20, 2012

[16] A.P.Ihom, J.I.Nor, O.O.Alabi, and A.W.Usman, ''Recent Trends and Developments in Surface Hardening Technology: A Focus on New Ceramics for Surface Modification in Industry,'' *International Journal of Science and Engineering Research*, (3)11, pp1-27, 2012.

[17] A.M.Shragger, *Elementary Metallurgy and Metallography*, 2ND Edition, Dover Publications New York, pp175-176, 1961.